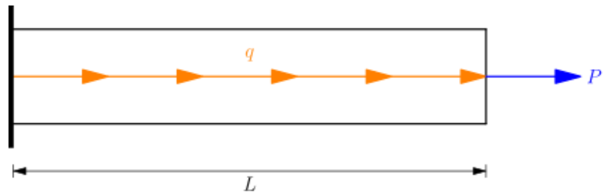
Ejemplo 1



Resolver

$$EA\,\frac{d^2u}{dx^2} + q = 0$$
$$u(0) = 0$$
$$u'(L) = \frac{P}{EA}$$

Solución exacta

$$u(x) = \frac{qL + P}{EA}x - \frac{q}{2EA}x^2$$

Solución aproximada generalizada

La forma débil de la ecuación diferencial es

$$\int_0^L R(x)\,W(x)\,dx = \int_0^L \left(EA\,\frac{d^2\hat{u}}{dx^2} + q\right)W\,dx = 0$$

multiplicando

$$\int_0^L W\,EA\frac{d^2\hat{u}}{dx^2}\,dx + \int_0^L W\,q\,dx = 0$$

Usando el teorema de Gauss o integrando por partes

$$\left(W\,EA\frac{d\hat{u}}{dx}\right)\Big|_0^L - \int_0^L \frac{dW}{dx}EA\frac{d\hat{u}}{dx}\,dx + \int_0^L W\,q\,dx = 0$$

Reordenando

$$\int_0^L \frac{dW}{dx}EA\frac{d\hat{u}}{dx}\,dx = \int_0^L W\,q\,dx + \left(W\,EA\frac{d\hat{u}}{dx}\right)\Big|_0^L$$

Reemplazando $F = EA\frac{d\hat{u}}{dx}$

$$\int_0^L \frac{dW}{dx}EA\frac{d\hat{u}}{dx}\,dx = \int_0^L W\,q\,dx + (W\,F)|_0^L$$