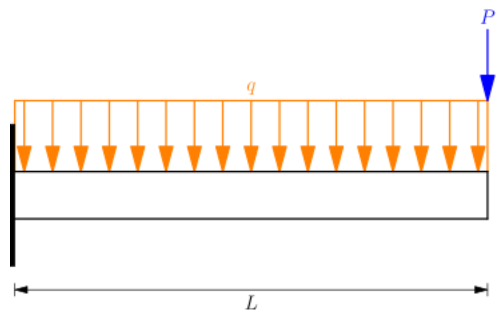
Ejemplo 6



Resolver

$$\begin{aligned} EI\,\frac{d^4v}{dx^4} + q = 0 \\ v(0) = 0 \qquad EIv''(L) = 0 \\ v'(0) = 0 \qquad EIv'''(L) = P \end{aligned}$$

Solución exacta

$$v(x) = -\frac{qL^2 + 2PL}{4EI}x^2 + \frac{qL + P}{6EI}x^3 - \frac{q}{24EI}x^4$$

Solución aproximada de quinto orden

La forma débil de la ecuación diferencial es

$$\int_0^L R(x)\,W(x)\,dx = \int_0^L \left(EI\,\frac{d^4\hat{v}}{dx^4} + q\right)W\,dx = 0$$

reduciendo el grado de las derivadas

$$\int_0^L \frac{d^2W}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W\,q\,dx - W(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$

usando bases de quinto orden en coordenadas locales

$$\begin{aligned} v(x) \approx \hat{v}(x) = {} & v_1\left(1 - \frac{23}{L^2}x^2 + \frac{66}{L^3}x^3 - \frac{68}{L^4}x^4 + \frac{24}{L^5}x^5\right) + \theta_1\left(x - \frac{6}{L}x^2 + \frac{13}{L^2}x^3 - \frac{12}{L^3}x^4 + \frac{4}{L^4}x^5\right) \\ & + v_2\left(\frac{16}{L^2}x^2 - \frac{32}{L^3}x^3 + \frac{16}{L^4}x^4\right) + \theta_2\left(-\frac{8}{L}x^2 + \frac{32}{L^2}x^3 - \frac{40}{L^3}x^4 + \frac{16}{L^4}x^5\right) \\ & + v_3\left(\frac{7}{L^2}x^2 - \frac{34}{L^3}x^3 + \frac{52}{L^4}x^4 - \frac{24}{L^5}x^5\right) + \theta_3\left(-\frac{1}{L}x^2 + \frac{5}{L^2}x^3 - \frac{8}{L^3}x^4 + \frac{4}{L^4}x^5\right) \end{aligned}$$

$\hat{v}_{xx}$ es

$$\begin{aligned} \frac{d^2\hat{v}}{dx^2} = {} & \left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right)v_1 + \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_1 \\ & + \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right)v_2 + \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right)\theta_2 \\ & + \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right)v_3 + \left(-\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_3 \end{aligned}$$

las funciones ponderadas son

$$W_1 = \frac{d\hat{v}}{dv_1} = 1 - \frac{23}{L^2}x^2 + \frac{66}{L^3}x^3 - \frac{68}{L^4}x^4 + \frac{24}{L^5}x^5$$
$$W_2 = \frac{d\hat{v}}{d\theta_1} = x - \frac{6}{L}x^2 + \frac{13}{L^2}x^3 - \frac{12}{L^3}x^4 + \frac{4}{L^4}x^5$$
$$W_3 = \frac{d\hat{v}}{dv_2} = \frac{16}{L^2}x^2 - \frac{32}{L^3}x^3 + \frac{16}{L^4}x^4$$
$$W_4 = \frac{d\hat{v}}{d\theta_2} = -\frac{8}{L}x^2 + \frac{32}{L^2}x^3 - \frac{40}{L^3}x^4 + \frac{16}{L^4}x^5$$
$$W_5 = \frac{d\hat{v}}{dv_3} = \frac{7}{L^2}x^2 - \frac{34}{L^3}x^3 + \frac{52}{L^4}x^4 - \frac{24}{L^5}x^5$$
$$W_6 = \frac{d\hat{v}}{d\theta_3} = -\frac{1}{L}x^2 + \frac{5}{L^2}x^3 - \frac{8}{L^3}x^4 + \frac{4}{L^4}x^5$$

formando el sistema de ecuaciones

$$\int_0^L \frac{d^2W_1}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W_1\,q\,dx - W_1(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W_1(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW_1(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW_1(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$
$$\int_0^L \frac{d^2W_2}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W_2\,q\,dx - W_2(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W_2(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW_2(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW_2(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$
$$\int_0^L \frac{d^2W_3}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W_3\,q\,dx - W_3(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W_3(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW_3(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW_3(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$
$$\int_0^L \frac{d^2W_4}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W_4\,q\,dx - W_4(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W_4(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW_4(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW_4(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$
$$\int_0^L \frac{d^2W_5}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W_5\,q\,dx - W_5(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W_5(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW_5(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW_5(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$
$$\int_0^L \frac{d^2W_6}{dx^2}\,EI\,\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W_6\,q\,dx - W_6(L)\,EI\frac{d^3\hat{v}(L)}{dx^3} + W_6(0)\,EI\frac{d^3\hat{v}(0)}{dx^3} + \frac{dW_6(L)}{dx}\,EI\frac{d^2\hat{v}(L)}{dx^2} - \frac{dW_6(0)}{dx}\,EI\frac{d^2\hat{v}(0)}{dx^2}$$

funciones ponderadas y sus derivadas

$$W_1 = 1 - \frac{23}{L^2}x^2 + \frac{66}{L^3}x^3 - \frac{68}{L^4}x^4 + \frac{24}{L^5}x^5 \qquad \frac{d^2W_1}{dx^2} = -\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3$$
$$W_2 = x - \frac{6}{L}x^2 + \frac{13}{L^2}x^3 - \frac{12}{L^3}x^4 + \frac{4}{L^4}x^5 \qquad \frac{d^2W_2}{dx^2} = -\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3$$
$$W_3 = \frac{16}{L^2}x^2 - \frac{32}{L^3}x^3 + \frac{16}{L^4}x^4 \qquad \frac{d^2W_3}{dx^2} = \frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2$$
$$W_4 = -\frac{8}{L}x^2 + \frac{32}{L^2}x^3 - \frac{40}{L^3}x^4 + \frac{16}{L^4}x^5 \qquad \frac{d^2W_4}{dx^2} = -\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3$$
$$W_5 = \frac{7}{L^2}x^2 - \frac{34}{L^3}x^3 + \frac{52}{L^4}x^4 - \frac{24}{L^5}x^5 \qquad \frac{d^2W_5}{dx^2} = \frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3$$
$$W_6 = -\frac{1}{L}x^2 + \frac{5}{L^2}x^3 - \frac{8}{L^3}x^4 + \frac{4}{L^4}x^5 \qquad \frac{d^2W_6}{dx^2} = -\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3$$

valores de las funciones ponderadas en los nodos

$$W_1(L) = 0 \quad W_1(0) = 1 \qquad \frac{dW_1(L)}{dx} = 0 \quad \frac{dW_1(0)}{dx} = 0$$
$$W_2(L) = 0 \quad W_2(0) = 0 \qquad \frac{dW_2(L)}{dx} = 0 \quad \frac{dW_2(0)}{dx} = 1$$
$$W_3(L) = 0 \quad W_3(0) = 0 \qquad \frac{dW_3(L)}{dx} = 0 \quad \frac{dW_3(0)}{dx} = 0$$

$$
\begin{aligned}
W_4(L) &= 0 \quad W_4(0) = 0 \qquad \frac{dW_4(L)}{dx} = 0 \quad \frac{dW_4(0)}{dx} = 0 \\
W_5(L) &= 1 \quad W_5(0) = 0 \qquad \frac{dW_5(L)}{dx} = 0 \quad \frac{dW_5(0)}{dx} = 0 \\
W_6(L) &= 0 \quad W_6(0) = 0 \qquad \frac{dW_6(L)}{dx} = 1 \quad \frac{dW_6(0)}{dx} = 0
\end{aligned}
$$

cortante y momento en los nodos

$$
\begin{aligned}
EI\frac{d^3\hat{v}(L)}{dx^3} &= V_3 \qquad EI\frac{d^3\hat{v}(0)}{dx^3} = V_1 \\
EI\frac{d^2\hat{v}(L)}{dx^2} &= M_3 \qquad EI\frac{d^2\hat{v}(0)}{dx^2} = M_1
\end{aligned}
$$

reemplazando

$$
\begin{aligned}
&\int_0^L \left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right) EI\Bigg[\left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right)v_1 \\
&+ \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_1 + \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right)v_2 + \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right)\theta_2 \\
&+ \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right)v_3 + \left(-\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_3\Bigg]\,dx = \\
&\qquad -\int_0^L \left(1 - \frac{23}{L^2}x^2 + \frac{66}{L^3}x^3 - \frac{68}{L^4}x^4 + \frac{24}{L^5}x^5\right) q\,dx - 0(V_3) + 1(V_1) + 0(M_3) - 0(M_1)
\end{aligned}
$$

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\begin{aligned}
&\int_0^L \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right) EI\Bigg[\left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right)v_1 \\
&+ \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_1 + \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right)v_2 + \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right)\theta_2 \\
&+ \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right)v_3 + \left(-\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_3\Bigg]\,dx = \\
&\qquad -\int_0^L \left(x - \frac{6}{L}x^2 + \frac{13}{L^2}x^3 - \frac{12}{L^3}x^4 + \frac{4}{L^4}x^5\right) q\,dx - 0(V_3) + 0(V_1) + 0(M_3) - 1(M_1)
\end{aligned}
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\begin{aligned}
&\int_0^L \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right) EI\Bigg[\left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right)v_1 + \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_1 \\
&+ \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right)v_2 + \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right)\theta_2 + \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right)v_3 \\
&+ \left(-\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_3\Bigg]\,dx = -\int_0^L \left(\frac{16}{L^2}x^2 - \frac{32}{L^3}x^3 + \frac{16}{L^4}x^4\right) q\,dx - 0(V_3) + 0(V_1) + 0(M_3) - 0(M_1)
\end{aligned}
$$

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\begin{aligned}
&\int_0^L \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right) EI\Bigg[\left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right)v_1 + \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_1 \\
&+ \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right)v_2 + \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right)\theta_2 + \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right)v_3 \\
&+ \left(-\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_3\Bigg]\,dx = -\int_0^L \left(-\frac{8}{L}x^2 + \frac{32}{L^2}x^3 - \frac{40}{L^3}x^4 + \frac{16}{L^4}x^5\right) q\,dx - 0(V_3) + 0(V_1) + 0(M_3) - 0(M_1)
\end{aligned}
$$

$$
\begin{aligned}
&\int_0^L \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right) EI\Bigg[\left(-\frac{46}{L^2} + \frac{396}{L^3}x - \frac{816}{L^4}x^2 + \frac{480}{L^5}x^3\right)v_1 + \left(-\frac{12}{L} + \frac{78}{L^2}x - \frac{144}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_1 \\
&+ \left(\frac{32}{L^2} - \frac{192}{L^3}x + \frac{192}{L^4}x^2\right)v_2 + \left(-\frac{16}{L} + \frac{192}{L^2}x - \frac{480}{L^3}x^2 + \frac{320}{L^4}x^3\right)\theta_2 + \left(\frac{14}{L^2} - \frac{204}{L^3}x + \frac{624}{L^4}x^2 - \frac{480}{L^5}x^3\right)v_3 \\
&+ \left(-\frac{2}{L} + \frac{30}{L^2}x - \frac{96}{L^3}x^2 + \frac{80}{L^4}x^3\right)\theta_3\Bigg]\,dx = -\int_0^L \left(\frac{7}{L^2}x^2 - \frac{34}{L^3}x^3 + \frac{52}{L^4}x^4 - \frac{24}{L^5}x^5\right) q\,dx - 1(V_3) + 0(V_1) + 0(M_3) - 0(M_1)
\end{aligned}
$$

$$\int_0^L \left(-\frac{2}{L}+\frac{30}{L^2}x-\frac{96}{L^3}x^2+\frac{80}{L^4}x^3\right)EI\left[\left(-\frac{46}{L^2}+\frac{396}{L^3}x-\frac{816}{L^4}x^2+\frac{480}{L^5}x^3\right)v_1+\left(-\frac{12}{L}+\frac{78}{L^2}x-\frac{144}{L^3}x^2+\frac{80}{L^4}x^3\right)\theta_1\right.$$
$$+\left(\frac{32}{L^2}-\frac{192}{L^3}x+\frac{192}{L^4}x^2\right)v_2+\left(-\frac{16}{L}+\frac{192}{L^2}x-\frac{480}{L^3}x^2+\frac{320}{L^4}x^3\right)\theta_2+\left(\frac{14}{L^2}-\frac{204}{L^3}x+\frac{624}{L^4}x^2-\frac{480}{L^5}x^3\right)v_3$$
$$\left.+\left(-\frac{2}{L}+\frac{30}{L^2}x-\frac{96}{L^3}x^2+\frac{80}{L^4}x^3\right)\theta_3\right]dx=-\int_0^L\left(-\frac{1}{L}x^2+\frac{5}{L^2}x^3-\frac{8}{L^3}x^4+\frac{4}{L^4}x^5\right)q\,dx-0(V_3)+0(V_1)+1(M_3)-0(M_1)$$

integrando

$$\frac{EI}{L^3}\left(\frac{5092}{35}v_1+\frac{1138L}{35}\theta_1-\frac{512}{5}v_2+\frac{384L}{7}\theta_2-\frac{1508}{35}v_3+\frac{242L}{35}\theta_3\right)=-\frac{7qL}{30}+V_1$$
$$\frac{EI}{L^3}\left(\frac{1138L}{35}v_1+\frac{332L^2}{35}\theta_1-\frac{128L}{5}v_2+\frac{64L^2}{7}\theta_2-\frac{242L}{35}v_3+\frac{38L^2}{35}\theta_3\right)=-\frac{qL^2}{60}-M_1$$
$$\frac{EI}{L^3}\left(-\frac{512}{5}v_1-\frac{128L}{5}\theta_1+\frac{1024}{5}v_2-\frac{512}{5}v_3+\frac{128L}{5}\theta_3\right)=-\frac{8qL}{15}$$
$$\frac{EI}{L^3}\left(\frac{384L}{7}v_1+\frac{64L^2}{7}\theta_1+\frac{256L^2}{7}\theta_2-\frac{384L}{7}v_3+\frac{64L^2}{7}\theta_3\right)=0$$
$$\frac{EI}{L^3}\left(-\frac{1508}{35}v_1-\frac{242L}{35}\theta_1-\frac{512}{5}v_2-\frac{384L}{7}\theta_2+\frac{5092}{35}v_3-\frac{1138L}{35}\theta_3\right)=-\frac{7qL}{30}-V_3$$
$$\frac{EI}{L^3}\left(\frac{242L}{35}v_1+\frac{38L^2}{35}\theta_1+\frac{128L}{5}v_2+\frac{64L^2}{7}\theta_2-\frac{1138L}{35}v_3+\frac{332L^2}{35}\theta_3\right)=\frac{qL^2}{60}+M_3$$

en forma matricial

$$\frac{EI}{L^3}\begin{bmatrix}\frac{5092}{35} & \frac{1138L}{35} & -\frac{512}{5} & \frac{384L}{7} & -\frac{1508}{35} & \frac{242L}{35}\\ \frac{1138L}{35} & \frac{332L^2}{35} & -\frac{128L}{5} & \frac{64L^2}{7} & -\frac{242L}{35} & \frac{38L^2}{35}\\ -\frac{512}{5} & -\frac{128L}{5} & \frac{1024}{5} & 0 & -\frac{512}{5} & \frac{128L}{5}\\ \frac{384L}{7} & \frac{64L^2}{7} & 0 & \frac{256L^2}{7} & -\frac{384L}{7} & \frac{64L^2}{7}\\ -\frac{1508}{35} & -\frac{242L}{35} & -\frac{512}{5} & -\frac{384L}{7} & \frac{5092}{35} & -\frac{1138L}{35}\\ \frac{242L}{35} & \frac{38L^2}{35} & \frac{128L}{5} & \frac{64L^2}{7} & -\frac{1138L}{35} & \frac{332L^2}{35}\end{bmatrix}\begin{bmatrix}v_1\\ \theta_1\\ v_2\\ \theta_2\\ v_3\\ \theta_3\end{bmatrix}=\begin{bmatrix}-\frac{7qL}{30}\\ -\frac{qL^2}{60}\\ -\frac{8qL}{15}\\ 0\\ -\frac{7qL}{30}\\ \frac{qL^2}{60}\end{bmatrix}+\begin{bmatrix}V_1\\ -M_1\\ 0\\ 0\\ -V_3\\ M_3\end{bmatrix}$$

reemplazando fuerzas y desplazamientos

$$\frac{EI}{L^3}\begin{bmatrix}\frac{5092}{35} & \frac{1138L}{35} & -\frac{512}{5} & \frac{384L}{7} & -\frac{1508}{35} & \frac{242L}{35}\\ \frac{1138L}{35} & \frac{332L^2}{35} & -\frac{128L}{5} & \frac{64L^2}{7} & -\frac{242L}{35} & \frac{38L^2}{35}\\ -\frac{512}{5} & -\frac{128L}{5} & \frac{1024}{5} & 0 & -\frac{512}{5} & \frac{128L}{5}\\ \frac{384L}{7} & \frac{64L^2}{7} & 0 & \frac{256L^2}{7} & -\frac{384L}{7} & \frac{64L^2}{7}\\ -\frac{1508}{35} & -\frac{242L}{35} & -\frac{512}{5} & -\frac{384L}{7} & \frac{5092}{35} & -\frac{1138L}{35}\\ \frac{242L}{35} & \frac{38L^2}{35} & \frac{128L}{5} & \frac{64L^2}{7} & -\frac{1138L}{35} & \frac{332L^2}{35}\end{bmatrix}\begin{bmatrix}0\\ 0\\ v_2\\ \theta_2\\ v_3\\ \theta_3\end{bmatrix}=\begin{bmatrix}-\frac{7qL}{30}\\ -\frac{qL^2}{60}\\ -\frac{8qL}{15}\\ 0\\ -\frac{7qL}{30}\\ \frac{qL^2}{60}\end{bmatrix}+\begin{bmatrix}V_1\\ -M_1\\ 0\\ 0\\ -P\\ 0\end{bmatrix}$$

resolviendo

$$v_2=-\frac{17qL^4+40PL^3}{384EI}$$
$$\theta_2=-\frac{7qL^3+18PL^2}{48EI}$$
$$v_3=-\frac{3qL^4+8PL^3}{24EI}$$
$$\theta_3=-\frac{qL^3+3PL^2}{6EI}$$

$$V_1 = qL + P$$
$$M_1 = -\frac{qL^2 + 2PL}{2}$$

reemplazando en la solución aproximada

$$\begin{aligned}
\hat{v}(x) &= \left(-\frac{17qL^4 + 40PL^3}{384EI}\right)\left(\frac{16}{L^2}x^2 - \frac{32}{L^3}x^3 + \frac{16}{L^4}x^4\right) + \left(-\frac{7qL^3 + 18PL^2}{48EI}\right)\left(-\frac{8}{L}x^2 + \frac{32}{L^2}x^3 - \frac{40}{L^3}x^4 + \frac{16}{L^4}x^5\right) \\
&\quad + \left(-\frac{3qL^4 + 8PL^3}{24EI}\right)\left(\frac{7}{L^2}x^2 - \frac{34}{L^3}x^3 + \frac{52}{L^4}x^4 - \frac{24}{L^5}x^5\right) + \left(-\frac{qL^3 + 3PL^2}{6EI}\right)\left(-\frac{1}{L}x^2 + \frac{5}{L^2}x^3 - \frac{8}{L^3}x^4 + \frac{4}{L^4}x^5\right) \\
&= -\frac{qL^2 + 2PL}{4EI}x^2 + \frac{qL + P}{6EI}x^3 - \frac{q}{24EI}x^4
\end{aligned}$$