# Introducción a elementos finitos
# Tarea 1 I-2016

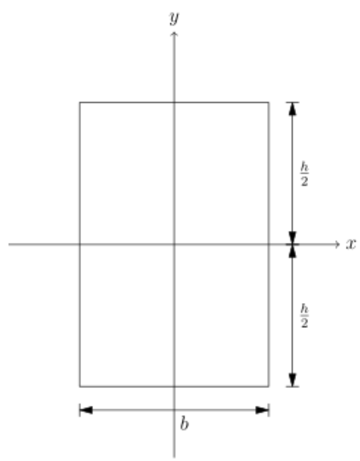
El factor de forma en general es

$$k = \int_A \frac{Q^2 A}{b^2 I^2}\, dA$$

si la sección es constante se transforma en

$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2}\, dA$$

## Sección rectangular




El momento estático es

$$Q = \int_A y\, dA = \int_y^{\frac{h}{2}} y\, b\, dy = b \int_y^{\frac{h}{2}} y\, dy$$

Integrando

$$Q = b \left. \frac{y^2}{2} \right|_y^{\frac{h}{2}} = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

Factor de forma

$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2}\, dA = \frac{A}{I^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{Q^2}{b^2}\, b\, dy = \frac{A}{bI^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} Q^2\, dy$$

Reemplazando valores

$$k = \frac{bh}{b\left(\frac{bh^3}{12}\right)^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\frac{b}{2}\left(\frac{h^2}{4} - y^2\right)\right]^2 dy$$

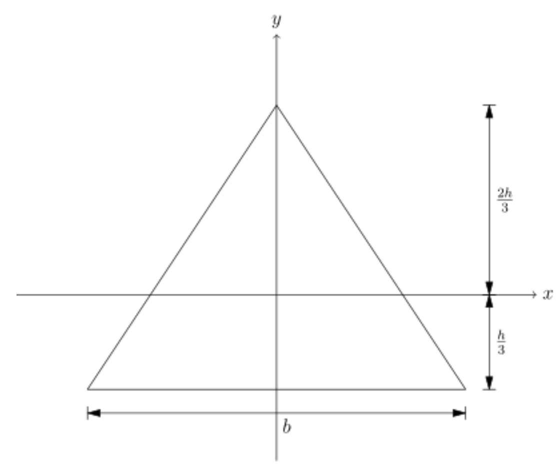
Integrando

$$k = \frac{36}{h^5}\left.\left(\frac{h^4}{16} y - \frac{h^2}{6} y^3 + \frac{1}{5} y^5\right)\right|_{-\frac{h}{2}}^{\frac{h}{2}}$$

Simplificando

$$k = \frac{36}{h^5}\left(\frac{h^5}{30}\right) = \frac{6}{5}$$

## Sección triangular



El momento estático es

$$Q = \int\limits_A y\ dA = \int_0^{\frac{2}{3}b-\frac{b}{h}y} \int_y^{\frac{2}{3}h-\frac{h}{b}x} y\ dy\ dx$$

Integrando respecto de $y$

$$Q = \int_0^{\frac{2}{3}b-\frac{b}{h}y} \left.\frac{y^2}{2}\right|_y^{\frac{2}{3}h-\frac{h}{b}x} dx = \int_0^{\frac{2}{3}b-\frac{b}{h}y} \frac{2h^2}{9} - \frac{y^2}{2} - \frac{2h^2}{3b}x + \frac{h^2}{2b^2}x^2\ dx$$

Integrando respecto de $x$

$$Q = \left[\left(\frac{2h^2}{9} - \frac{y^2}{2}\right)x - \frac{h^2}{3b}x^2 + \frac{h^2}{6b^2}x^3\right]\Bigg|_0^{\frac{2}{3}b-\frac{b}{h}y} = \frac{4bh^2}{81} - \frac{b}{3}y^2 + \frac{b}{3h}y^3$$

Factor de forma

$$k = \frac{A}{I^2}\int\limits_A \frac{Q^2}{b^2}\ dA = \frac{A}{I^2}\int_{-\frac{h}{3}}^{\frac{2h}{3}} \frac{Q^2}{x^2}\ x\ dy = \frac{A}{I^2}\int_{-\frac{h}{3}}^{\frac{2h}{3}} \frac{Q^2}{x}\ dy$$

Reemplazando valores

$$k = \frac{\frac{bh}{2}}{\left(\frac{bh^3}{36}\right)^2}\int_{-\frac{h}{3}}^{\frac{2h}{3}} \frac{\left(\frac{4bh^2}{81} - \frac{b}{3}y^2 + \frac{b}{3h}y^3\right)^2}{\frac{2}{3}b - \frac{b}{h}y}\ dy$$

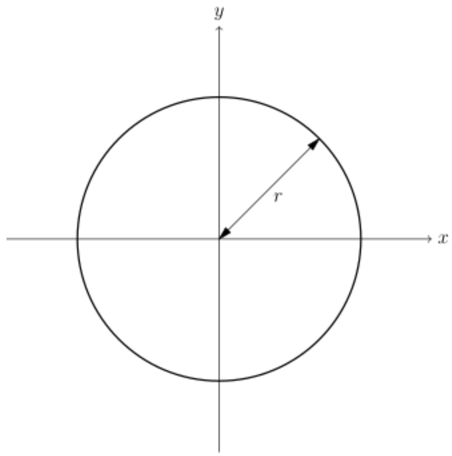
Integrando

$$k = \frac{648}{bh^5}\left(\frac{8bh^4}{2187}y + \frac{2bh^3}{729}y^2 - \frac{10bh^2}{729}y^3 - \frac{bh}{324}y^4 + \frac{4b}{135}y^5 - \frac{b}{54h}y^6\right)\Bigg|_{-\frac{h}{3}}^{\frac{2h}{3}}$$

Simplificando

$$k = \frac{648}{bh^5}\left(\frac{bh^5}{540}\right) = \frac{6}{5}$$

## Sección circular




El momento estático es

$$Q = \int\limits_A y\ dA = 2\int_0^{\sqrt{r^2-y^2}} \int_y^{\sqrt{r^2-x^2}} y\ dy\ dx$$

Integrando respecto de $y$

$$Q = 2\int_0^{\sqrt{r^2-y^2}} \left.\frac{y^2}{2}\right|_y^{\sqrt{r^2-x^2}} dx = 2\int_0^{\sqrt{r^2-y^2}} r^2 - y^2 - x^2\ dx$$

Integrando respecto de $x$

$$Q = \left[\left(r^2 - y^2\right)x - \frac{1}{3}x^3\right]\Bigg|_0^{\sqrt{r^2-y^2}} = \frac{2}{3}\left(r^2 - y^2\right)^{\frac{3}{2}}$$

Factor de forma

$$k = \frac{A}{I^2}\int\limits_A \frac{Q^2}{b^2}\ dA = \frac{A}{I^2}\int_{-r}^{r} \frac{Q^2}{(2x)^2}\ 2x\ dy = \frac{A}{I^2}\int_{-r}^{r} \frac{Q^2}{2x}\ dy$$

Reemplazando valores

$$k = \frac{\pi r^2}{\left(\frac{\pi r^4}{4}\right)^2} \int_{-r}^{r} \frac{\left[\frac{2}{3}\left(r^2 - y^2\right)^{\frac{3}{2}}\right]^2}{2\sqrt{r^2 - y^2}}\ dy$$

Simplificando

$$k = \frac{32}{9\pi r^6} \int_{-r}^{r} \left(r^2 - y^2\right)^{\frac{5}{2}}\ dy$$

Integrando

$$k = \frac{32}{9\pi r^6} \left[\frac{33r^4 y - 26r^2 y^3 + 8y^5}{48} \sqrt{r^2 - y^2} + \frac{5r^6}{16} \arcsin\left(\frac{y}{r}\right)\right]\Bigg|_{-r}^{r}$$

Simplificando

$$k = \frac{32}{9\pi r^6}\left(\frac{5\pi r^6}{16}\right) = \frac{10}{9}$$

# Referencias

[1] Hamid Rezayekhadjavi. Shear form factors for various cross-sections. 1979.