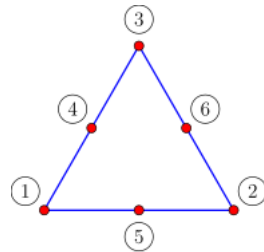


Introducción a elementos finitos

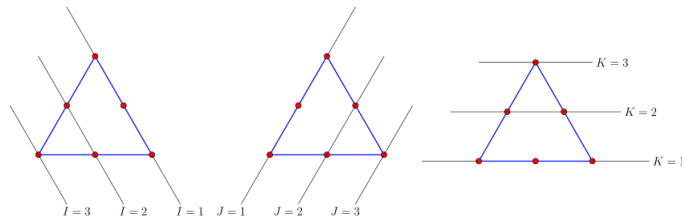
Tarea 5 I-2016

Calcular las funciones de forma para el elemento triangular cuadrático o LST



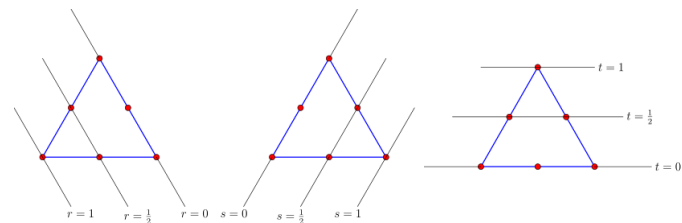
Numeración

Numeración de nodos



$$\begin{aligned} \textcircled{1} &= [I_1, J_1, K_1] = [3, 1, 1] & \textcircled{4} &= [I_4, J_4, K_4] = [2, 1, 2] \\ \textcircled{2} &= [I_2, J_2, K_2] = [1, 3, 1] & \textcircled{5} &= [I_5, J_5, K_5] = [2, 2, 1] \\ \textcircled{3} &= [I_3, J_3, K_3] = [1, 1, 3] & \textcircled{6} &= [I_6, J_6, K_6] = [1, 2, 2] \end{aligned}$$

Coordenadas de nodos



$$\begin{aligned} \textcircled{1} &= [r_3, s_1, t_1] = [1, 0, 0] & \textcircled{4} &= [r_2, s_1, s_2] = \left[\frac{1}{2}, 0, \frac{1}{2}\right] \\ \textcircled{2} &= [r_1, s_3, t_1] = [0, 1, 0] & \textcircled{5} &= [r_2, s_2, t_1] = \left[\frac{1}{2}, \frac{1}{2}, 0\right] \\ \textcircled{3} &= [r_1, s_1, t_3] = [0, 0, 1] & \textcircled{6} &= [r_1, s_2, t_2] = \left[0, \frac{1}{2}, \frac{1}{2}\right] \end{aligned}$$

Nodo ①

Reemplazando numeración y coordenadas

$$\begin{aligned} T_3(r) &= \frac{r - r_2}{r_3 - r_2} \cdot \frac{r - r_1}{r_3 - r_1} = \frac{r - \frac{1}{2}}{1 - \frac{1}{2}} \cdot \frac{r - 0}{1 - 0} = r(2r - 1) \\ T_1(s) &= 1 \\ T_1(t) &= 1 \end{aligned}$$

Reemplazando polinomios

$$N_1 = T_3 T_1 T_1 = r(2r - 1) \cdot 1 \cdot 1 = r(2r - 1)$$

Nodo ②

Reemplazando numeración y coordenadas

$$\begin{aligned} T_1(r) &= 1 \\ T_3(s) &= \frac{s - s_2}{s_3 - s_2} \cdot \frac{s - s_1}{s_3 - s_1} = \frac{s - \frac{1}{2}}{1 - \frac{1}{2}} \cdot \frac{s - 0}{1 - 0} = s(2s - 1) \\ T_1(t) &= 1 \end{aligned}$$

Reemplazando polinomios

$$N_2 = T_1 T_3 T_1 = 1 \cdot s(2s - 1) \cdot 1 = s(2s - 1)$$

Nodo ③

Reemplazando numeración y coordenadas

$$T_1(r) = 1$$

$$T_1(s) = 1$$

$$T_3(t) = \frac{t - t_2}{t_3 - t_2} \cdot \frac{t - t_1}{t_3 - t_1} = \frac{t - \frac{1}{2}}{1 - \frac{1}{2}} \cdot \frac{t - 0}{1 - 0} = t(2t - 1)$$

Reemplazando polinomios

$$N_3 = T_1 T_1 T_3 = 1 \cdot 1 \cdot t(2t - 1) = t(2t - 1)$$

Nodo ④

Reemplazando numeración y coordenadas

$$T_2(r) = \frac{r - r_1}{r_2 - r_1} = \frac{r - 0}{\frac{1}{2} - 0} = 2r$$

$$T_1(s) = 1$$

$$T_2(t) = \frac{t - t_1}{t_2 - t_1} = \frac{t - 0}{\frac{1}{2} - 0} = 2t$$

Reemplazando polinomios

$$N_4 = T_2 T_1 T_2 = 2r \cdot 1 \cdot 2t = 4rt$$

Nodo ⑤

Reemplazando numeración y coordenadas

$$T_2(r) = \frac{r - r_1}{r_2 - r_1} = \frac{r - 0}{\frac{1}{2} - 0} = 2r$$

$$T_2(s) = \frac{s - s_1}{s_2 - s_1} = \frac{s - 0}{\frac{1}{2} - 0} = 2s$$

$$T_1(t) = 1$$

Reemplazando polinomios

$$N_5 = T_2 T_2 T_1 = 2r \cdot 2s \cdot 1 = 4rs$$

Nodo ⑥

Reemplazando numeración y coordenadas

$$T_1(r) = 1$$

$$T_2(s) = \frac{s - s_1}{s_2 - s_1} = \frac{s - 0}{\frac{1}{2} - 0} = 2s$$

$$T_2(t) = \frac{t - t_1}{t_2 - t_1} = \frac{t - 0}{\frac{1}{2} - 0} = 2t$$

Reemplazando polinomios

$$N_6 = T_1 T_2 T_2 = 1 \cdot 2s \cdot 2t = 4st$$